smaller values of M, the acceleration during the first phase is maximum and the result is

$$\Delta V = 2(gD)^{1/2}\left[\frac{2(1 + M^2)^{1/2} - 1}{M}\right]^{1/2} \quad (18)$$

Equations (17) and (18) are plotted in Fig. 3.

The horizontal translation itself forms a subclass with respect to all possible maneuvers between the required end conditions. The best such maneuver (two impulses) results in $\Delta V/(gD)^{1/2} = 2.0$. The fuel penalty for restricting the maneuver to be horizontal is thus about 25%.

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Technique for Minimizing the Specific Weight of a Finned Beryllium Space Radiator

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Nomenclature

B = total width of tube with fins, ft
C = specific heat, Btu/lb, °R
D = tube diameter, ft
ϵ = emissivity
k = thermal conductivity, Btu/hr, ft, °R
K = constant
l = length along tube, ft.
L = tube length, ft
Q_r = $f(V)$ = rejected power, Btu/hr
t = fin thickness, ft
$\bar{T}$ = temperature, °R
T_w = temperature of tube wall, °R
T = average radiating temperature, °R
T_{w1} = temperature of tube wall at tube inlet, °R
T_{w_2} = temperature of tube wall at tube outlet, °R
V = velocity of liquid metal in tube, ft/hr
W = $F(V)$ = total weight, lb_f
X = length from center of tube to tip of fin, ft
σ = Stefan-Boltzmann constant, Btu/hr, ft^2, °R^4
ρ = density of liquid metal, lb/ft^3

SINCE in studying large spacecraft power plants, it has been found that the radiator constitutes a major portion of the total weight of the powerplant,[1] the importance of minimizing the specific weight of the radiator is apparent. Beryllium is a very attractive material for high-temperature, finned space radiators.[2] Reference 2 also indicates that tube diameter, fin thickness, and length should be small so that the specific weight (lb/kw of heat rejected) of the radiator may be low. Reference 2 deals with the rectangular fin and assumes that the thermal conductivity of the fin material is independent of the temperature. Reference 3 indicates that a triangular fin is more effective in rejecting heat than a rectangular one, and hence such a fin is better suited for a space radiator.

This article, therefore, discusses a beryllium radiator having a fin that is triangular in cross section and containing a wettable liquid metal in the tube for power plant cooling, and it presents a simple technique for minimizing the specific weight of a small-diameter beryllium tube with a short and thin triangular fin. This technique permits thermal conductivity to vary with temperature and can be used for any fin profile. It also allows the temperature of the base of the fin to vary, accounts for the weight of the radiator fluid and pump, and includes the effect of these parameters in the optimization of the specific weight of the radiator.

The principal problem is to express the heat rejected by the radiator as a function of the fluid velocity, since the combined weight of the radiator and pump can be expressed as a function of the fluid velocity by fundamental and well-known equations. By using the generally accepted value of 0.8 for emissivity of the radiator surface, it is shown that the relationship between the temperature of the tube wall and the average radiating temperature of the finned tube at temperatures above 800°R is almost linear. If, as a good approximation, this relationship is assumed to be linear, the equation for the specific weight of the radiator becomes quite simple and its minimum point can be found easily.

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Assumptions

In treating this subject, the following assumptions have been made: 1) the radiating surface is a gray surface; 2) the diameter of the tube is such that the temperature of the tube around the circumference is constant, and the radiant interaction between fin and tube is negligible; 3) heat flow by conduction is one-dimensional; 4) resistance to the flow of heat from the liquid metal to the outer surface of the tube is negligible; 5) the radiating area is a flat surface, and any energy incident on it is not considered.

Analysis

The energy lost by the liquid metal as it passes through the finned tube is

$$Q_r = \frac{\pi D^2 V\rho C(T_{W_1} - T_{W_2})}{4} \quad (1)$$

An energy balance for the nodal point 1 in Fig. 1 gives

$$Q = \frac{k_{0,1}t_{0,1}l}{\Delta X}(T_W - T_1) - \frac{k_{1,2}t_{1,2}l}{\Delta X}(T_1 - T_2) - 2\sigma\epsilon\Delta X\, lT_1^4 \quad (2)$$

Equations are set up for the other nodal points. In the steady state, Q is equal to zero, and the unknown temperatures T_1, T_2, T_3, and T_4 are solved by relaxation methods. The values of T_W and ϵ are fixed, but that of k varies with temperature. By using the typical temperature curve in Fig. 1, one finds that the energy balance is

$$\sigma\epsilon l\Delta X(T_1^4 + T_2^4 + \ldots T_4^4) + \frac{\sigma\epsilon lDT_W^4}{2} = \sigma\epsilon Xl\bar{T}^4 \quad (3)$$

The average radiating temperature $\bar{T}$ at any position of the finned tube is obtained from this equation. By varying the value of T_W, Eqs. (2) and (3) give the relationship between T_W and $\bar{T}$ for a particular fin and tube (see Fig. 2). The tube diameter is $\frac{1}{16}$ in., the emissivity is 0.8, and the fins have a triangular cross section with a root thickness of 0.004 in. The length of the fin varies from 0.5 to 2.0 in. Similar curves were obtained for tubes having diameters of $\frac{1}{8}$ in. and $\frac{1}{4}$ in., with a fin root thickness of 0.008 in. and the lengths indicated above. From Fig. 2, $dT_w/d\bar{T}$ can be considered constant for a particular curve. The energy balance for an infinitesimal element of the finned tube is

$$-\pi D^2 V\rho C(dT_W)/4 = 2\sigma\epsilon B(dl)\bar{T}^4 \quad (4)$$

Since $dT_W/d\bar{T} = K =$ constant, integrating and solving for $\bar{T}$ in Eq. (4) gives

$$\bar{T} = \left(\frac{\bar{T}_1^3\pi D^2 V\rho CK}{\pi D^2 V\rho CK + 24\sigma\epsilon BL\bar{T}_1^3}\right)^{1/3} \quad (5)$$

Integrating $dT_W/d\bar{T} = K$ gives

$$T_{W_1} - T_{W_2} = K(\bar{T}_1 - \bar{T}) \quad (6)$$

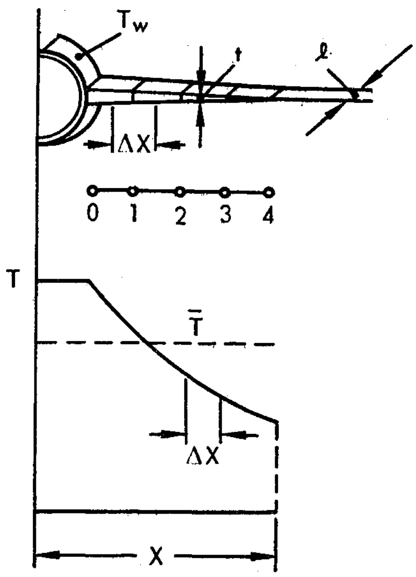

Fig. 1 Radiator element with temperature distribution and average radiating temperature.

Combining Eqs. (1), (5), and (6) yields

$$Q_r = \frac{\pi D^2 V \rho C K}{4}\left[\bar{T}_1 - \left(\frac{\bar{T}_1{}^3 \pi D^2 V \rho C K}{\pi D^2 V \rho C K + 24\sigma\epsilon B L \bar{T}_1{}^3}\right)^{1/3}\right] \qquad (7)$$

The weight of the radiator and pump is a function of the fluid velocity. This weight is the sum of the weights of the tube, fin, liquid metal, and pump. For a particular radiator, the weights of the fin and liquid metal are fixed. The pump weight is calculated from the pumping power and the specific pump weight in lb/kw of pumping power. The weight of the tube and the pumping power are expressed as a function of fluid velocity by fundamental equations since pressure drop as a function of fluid velocity is expressed by the fundamental equation for incompressible flow. Thus, for a particular radiator and pump, the weight and heat rejected are functions $F(V)$ and $f(V)$, respectively, of the fluid velocity. The specific weight of the radiator is then

$$W_S = W/Q_r = F(V)/f(V) \qquad (8)$$

By using the specific pump weight as a parameter, a set of curves with minimum points for the specific weight of the radiator can be constructed from Eq. (8) by varying the fluid velocity.

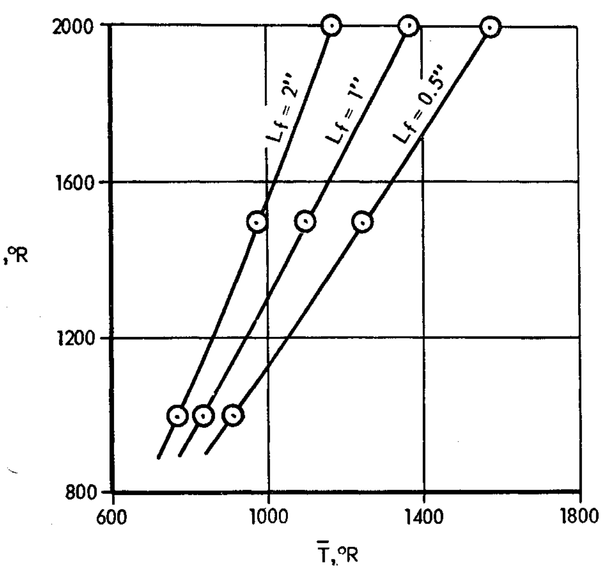

Fig. 2 Average radiating temperature of fin plus tube vs tube wall temperature (emissivity = 0.8).

Conclusions

The technique presented is of value to designers working with liquid-metal, finned beryllium space radiators since it permits a quite rapid optimization calculation of the radiator without the necessity of solving complicated differential equations. The calculated thickness of the tube wall can be increased in order to meet reliability requirements with respect to meteoroid puncture with negligible heat transfer effect since beryllium has good thermal conductivity.

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Hypersonic Wake Characteristics behind Spheres and Cones

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Growth of Turbulent Wake

From measurements made from a simultaneously exposed series of shadowgraph photographs of the wake, the width of the turbulent core of the wake δ was determined as a function of x, the downstream distance behind the projectile. Since the edge of the turbulent core is irregular, an averaging process for the determination of the core width was necessary. In this case, the averaging was achieved by tracing the boundary of a 7-in. length of the shadowgraph of the turbulent core with a planimeter. Then the core width δ was obtained by dividing the planimeter reading by the 7-in. length. At the ambient range pressures used (about 40 to 200 mm Hg), no observations were possible with shadowgraph instrumentation for about the first 200 diam behind spheres. However, behind cones the viscous core was clearly visible even at an ambient range pressure of 20 mm Hg.

It has been suggested in Ref. 1 that for incompressible flow the width of the turbulent core δ is proportional to the cube root of (xC_DA), where C_D is the drag coefficient of the body and A the area on which the drag coefficient is based. In an equation form, this can also be written as

$$(\delta/d)/C_D{}^{1/3} = K(x/d)^{1/3} \qquad (1)$$

where d is the body diameter and K a constant of proportionality. Figure 1 shows a log-log plot of $(\delta/d)C_D{}^{-1/3}$ vs x/d of the Naval Ordnance Laboratory results for both spheres and cones. Also shown in the figure are the freestream Mach numbers M_∞ and freestream Reynolds numbers based on body diameter $R_{\infty d}$, at which the data were obtained. For the spheres, a constant drag coefficient of 0.9 was used. The drag coefficients for the 8° half-angle cones, including induced pressure and transverse curvature effects, were theoretically computed.[2] As can be seen from Fig. 1, when nondimensionalized in this fashion, the widths of the turbulent core of the wake behind both blunt and slender bodies collapse on a single curve. The equation of the curve which best describes the experimental results is

$$(\delta/d)/C_D{}^{1/3} = 0.9(x/d)^{1/3} \qquad (2)$$

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